STATISTICAL EVALUATION OF WAVE CONDITIONS IN A DELTAIC AREA

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SUMMARY

Coastal engineering problems concerning wind waves and swell can be solved with the aid of hydraulic or mathematical models, The irregular wave field i.e. the state of the sea surface can be described in a sufficient way for engineering problems either by parameters such as significant wave height and mean wave period, or in the form of power spectrum function and the zero-moment thereof.

A semi-empirical method is developed using transfer functions in order to determine the boundary conditions from wave measurements on a limited number of stations in all important points within a shallow sea area.

An economical design is usually possible if the probability of occurrence of all parameters concerned is known. The extrapolation of multidimensional statistical distributions of such parameters is often based on a relatively short period of field observations. The accuracy of the conclusions drawn from these observations influence the methods applied in the modelstudies and the reliability of the economical decision.

In this respect, an analysis of the available data is made with reference to some engineering and navigational problems in the South-Eastern part of the North Sea.

1. INTRODUCTION

A practical solution for maritime and coastal engineering problems, derived from the results of studies based on uniform waves, was only possible in some exceptional cases. Consequently it was necessary to initiate the studies of irregular waves for which windflumes are a necessary tool; as most of the hydraulic laboratories have installed now.

An important improvement is the introduction of a fetch-independant generator of irregular waves. Except for three dimensional problems, most of the maritime structures, breakwaters and dikes can be designed by using this new laboratory facility.

The accuracy of present designs is mainly determined by the knowledge of adequate boundary conditions based on the statistical analysis of wave observations.

The following aspects of the statistical analysis of wave readings in a deltaic area are discussed:

- The definition of a limited number of statistical parameters describing a field of irregular waves given as a time series.
- The determination of the probability distribution of the parameters based on a large number of time series observations.

The determination of the boundary conditions at all points of interest within a deltaic area where the wave data are known for a limited number of reference stations only.

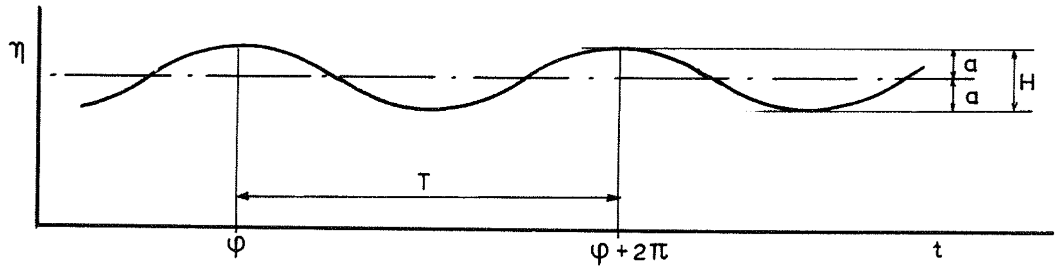
Two examples of the application of the statistical evaluations of the wave conditions to some problems in marine and coastal engineering are given.

In this general dealing with these examples the description of the method of application prevails over the details of the results.

2. DEFINITION OF SOME TERMS

2.1. Boundary conditions are the parameters or a system of parameters which are a basis for the determination of the design of a structure. Usually it is a multidimensional function of parameters comprising sea level, wave height and wave period. Very often, the mean wave direction is of importance. For different values of probability of exceedance of the parameters, studies in hydraulic models or computations have to be carried out in order to obtain physical data for the mathematical decision (Ref.1). For many problems the energy density spectrum function must be introduced replacing both wave height and wave period.

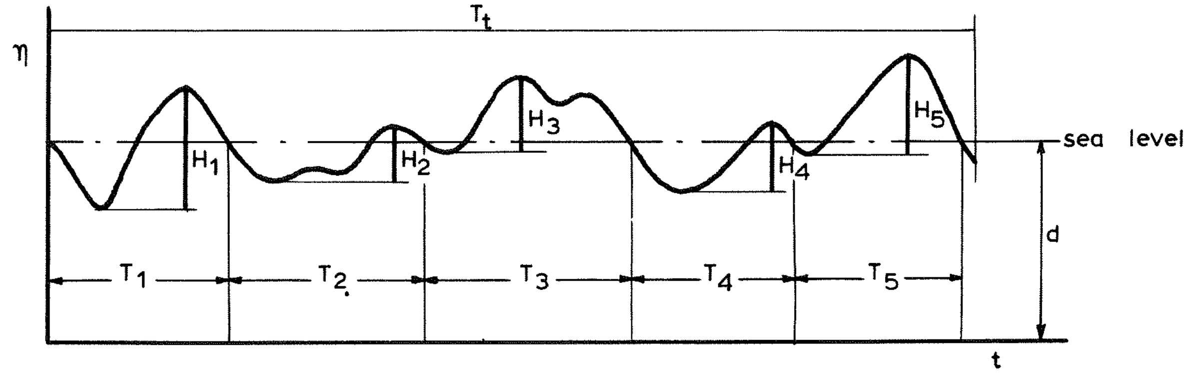
2.2. Uniform periodical waves, first order wave theory



movement of the water level as a function of time t	$\eta = a \cos(\omega t - \varphi)$
amplitude	a
wave height	$H = 2a$
wave period i.e. distance between two maxima of	T
angular velocity	$\omega = \frac{2\pi}{T}$
wave frequency	$f = \frac{1}{T}$
wave phase	φ
water depth	d
wave celerity (propagation velocity of the wave form)	$C = C(\omega, d)$
wave energy per unity of sea surface	$E = \frac{1}{8} \cdot \rho g H^2$

energy propagation velocity (group celerity)	$C_g = n.C$ with $\frac{1}{2} \leq n(\omega, d) \leq 1$
energy transport (wave power)	$N = E.C_g$

2.3. Irregular wave field



movement of the water level (time series)	$\eta = \eta(t)$
length of the time series	T_t
individual period	T_i = distance between two crossings of the sea level for $\frac{d\eta}{dt} < 0$
individual wave height	H_i = distance between the maximum and minimum of η within the corresponding period T_i
number of individual waves	n
mean period	$T_m = \frac{T_t}{n}$
representative period	T_r
significant wave height	H_s
wave height corresponding with exceedance value q	H_q
exceedance probability q	$q(Hq)$=percentage of the number of individual wave heights which are higher than Hq.
frequency of "wave components" in a wave field	f
autocorrelation function	$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{1}{2}T}^{+\frac{1}{2}T} \eta(t).\ t+\tau)dt$
** energy density spectrum (total energy)	$S(f) = \int_{-\infty}^{+\infty} R(\tau) e^{2\pi i f \tau} d\tau = 2\int_0^{\infty} R(\tau).\cos 2\pi f\tau.d\tau$
potential energy of a time series	$E_p = \rho g m_o = \rho g \int_0^{\infty} S(f)\, df$
** zero moment of the real part of spectrum	$m_o = \int_0^{\infty} S(f) df$
total energy of wave field	$E_T = 2E_p = 2 \rho g m_o$

3. STATISTICAL PARAMETERS DESCRIBING A WAVE FIELD

3.1. General approach

Most of the parameters which are generally used in order to define the wave movement in fluids are derived from the theory of periodical uniform waves. This theory, in which the mathematical relations between the parameters are determined, could be applied to many kinds of sharply defined and schematized problems. In deep water, the solution of the linearized differential equations is correct for first order harmonic waves of small amplitude. A number of solutions of non-linear equations exists for the waves of a special type in shallow water i.e. the trochoid wave, the third and higher order waves, the cnoidal wave etc. (Ref.2,3 and 4). The results of such theoretical studies are more or less verified by the results obtained by hydraulic model research in wave flumes using periodical wave generators. Such studies have contributed to the improvement of knowledge of the basic wave movement mechanism.

Many times, however, only approximate and sometimes even wrong solutions of engineering problems are obtained from the mathematical and model studies based on uniform waves. Moreover the variability of the seawaves limits the use of the very refined results of the uniform wave theory.

For irregular wave fields, the parameters defined for uniform waves, are to be expressed in statistical terms such as the significant wave height H_s, mean period T_m or reference wave celerity C_r. These parameters determined by empirical or semi-empirical distribution functions describe the behaviour of the "individual waves" of a wave field. However, an "individual wave" can only momentarily be defined. In fact it is a sum of many random disturbances travelling with different celerities. Obviously the mathematical relations between the different parameters of uniform waves does not hold for the "individual waves". Some times these relations may be used as a good approximation for the relations between the statistical parameters describing the wave field. The reliability of the results, however, depends on the proper choice amongst them.

The development in the field of the communication theory and especially the analysis of time series introduced by RICE in 1944 (Ref.5) opened new possibilities for study of the irregular wave fields. Until about 1957 the new statistical approach was mainly explored by oceanographers and mathematiciens.

The most important improvements in the application of the theory of irregular wave fields on engineering projects are the introduction of the spectral analysis of wave recordings and the Rayleigh-distribution function of wave heights based on the first order harmonic theory. (Ref.6).

3.2. The energy density spectrum is obtained from the auto-correlation analysis of wave records. It gives information about the potential energy $\rho g \frac{\Delta E}{\Delta f}$ within a narrow range of wave frequenties Δf as a function of the corresponding frequency f. Usually, this energy is divided by $\rho . g$ and has the dimensions of m^2

* $$\left(\frac{\Delta E}{\Delta f}\right)_j = F(f_{j+\frac{1}{2}}) \text{ for } j = 1,2,3, \ldots\ldots.. \quad (1)$$

If $\Delta f \rightarrow o$, than $\frac{\Delta E}{\Delta f} \rightarrow \frac{dE}{df} = S(f)$ which is called the continuous ** **energy density spectrum**. In fact it is the real part of the fourier-transform of the autocorrelation function (Ref.2).

The energy density function is defined for stationary processes of long time series. The spectral analysis of times series offers the possibility for approximation of the irregular sea surface in a flume by means of the spectral components representing the energy in every class $(j+\frac{1}{2})$ of the frequency f. Two methods which are directly based on the energy density spectrum for the boundary condition in a flume are discussed in paper no.2 of this symposium.

The accuracy of this approach is better then the accuracy of the generation of waves in wind flumes depending on fetch. It is, however, still limited on account of the statistical character of the spectrum which is derived from a limited time series and usually represents only one sample. In other words the spectrum function does not determine the wave motion uniquelly.

It is important to note, that the definition of the total energy of spectra opens the possibility to determine the relation between the irregular wave motion and the uniform wave theory:

The total energy E_u of an uniform sinusoidal wave field per unity of surface may be expressed by the energy-height H_u of the uniform sinus wave:

$$E_u = \frac{1}{8} H_u^2 \qquad [m^2] \quad (2)$$

The energy represented by the high frequency components of a spectrum tends to zero. For practical purposes the values of $j.\Delta f$ may be limited to $f_{max} = (n+\frac{1}{2}).\Delta f < 5$ Hz.

Consequently, the total energy of the wave field may be approximated by the energy of uniform sinusoidal wave components computed for n classes of $f(j+\frac{1}{2})$.

** $$E_t = \rho g \sum_{j=o}^{j=n} 2\left(\frac{\Delta E}{\Delta f}\right)_j = \rho g \sum_{i=o}^{i=n} \frac{1}{8} H_i^2 = \frac{1}{8} \rho g H_s^2 \quad (3)$$

where H_i = individual wave height and

H_s = significant wave height, which is the wave height of an uniform sinus-wave with the same energy as the wave field.

In consequence,

$$H_s^2 = 16 \sum_{j=o}^{j=n} \left(\frac{\Delta E}{\Delta f}\right)_j \quad (4)$$

and for f o

$$H_s^2 = 16 \int_o^{f_{max}} S(f)\, df = 16\, m_o \quad (5)$$

in which m_o = zero moment of the real part of the spectral function.

Finally, it follows:

$$H_s = 4\sqrt{m_o} \qquad (6)$$

3.3. The height of the "individual wave"

The statistical probability distributions of the different parameters related to the "individual waves" are a reliable basis for the comparison between different records, except for the extreme values in the low probability range. The practical treatment of this problem is dealt with here in after, by discussing the probability distribution function of the "individual wave heights".

As mentioned before, the mathematical approach of LONGUET-HIGGINS results in the Rayleigh probability distribution of wave heights. In an other publication, CARTWRIGHT and LONGUET-HIGGINS (Ref.7) show the relation of the Rayleigh function with a sharply peaked spectrum of which the energy is concentrated in a single narrow frequency band.

By this relation they show, that the individual wave height H_r which is representative for the total energy of the narrow spectrum equals:

$$H_r = 4\sqrt{m_o} \qquad (7)$$

and the statistical parameter H_r of an irregular wave field appears to be equal to the sinusoidal wave height H_s determined from the relation between the energy of the uniform sinus wave and the total energy of the wave field.

The cumulative probability distribution is given by BRETSCHNEIDER (Ref.8) in the equation:

$$* \quad q\left(\frac{H_q}{\bar{H}}\right) = 1 - \exp.\left\{-\frac{\pi}{4}\left(\frac{H_q}{\bar{H}}\right)^2\right\} \qquad (8)$$

in which $\bar{H}$ = the "average" wave height

H_q = wave height with the cumulative probability value q. (Exceedance probability value = 1-q)

* $\bar{H}$ is related to the significant wave height as $\bar{H} = 0.625\ H_s$

Note, that this equation is expressed in dimensionless parameters. The exceedance probability of all other "individual" wave heights, including the representative wave height e.g. of the significant wave height can be computed from equation (8), on the condition that the assumptions of LONGUET-HIGGINS are valid. Moreover, the time series must remain stationary for a sufficiently long duration.

The extrapolation of the cumulative probability function to the extremely high waves is only correct for a limited time series when a range of significancy is considered (fig.3.1), as shown by GUMBEL (Ref.9). For the research in behalf of an engineering project the stationary process is often limited to the duration of a severe gale, which often are lasting only a few hours.

CARTWRIGHT and LONGUET-HIGGINS developed also a different category of probability functions by introducing a parameter describing the shape of the spectrum.

In the authors contention the practical value of theoretical refinements of the statistical analysis of timeseries is limited by the uncertainities of the extreme values mentioned by GUMBEL. This opinion holds also for the determination of the probability distributions of wave heights in shallow water with respect to the effect of the non-linear inter-action between the wave components.

The wave readings of limited duration, e.g. 30 min, at some stations in the shallow parts of the North Sea show only small deviations of the extreme values of wave heights from the Rayleigh-distribution. The wave heights at the exceedance probability level of 0,5% are almost never larger then 15% and/or smaller then 10% of the corresponding wave heights obtained from the Rayleigh-distribution. The Rayleigh distribution appears to be acceptable in shallow water as far as in the breaker zone (fig.3.2).

Moreover, there is an empirical relation between the significant wave height and the actual water depth:

$$H_s = (0,4 \pm 0,05)\ d \qquad (9)$$

This relation is of importance for the determination of the boundary conditions near the coast and in the deltaic areas where the actual depth usually depends on both the tidal movement and the wind effect during storms. Consequently, the boundary conditions for waves in this areas often are determined by these two factors.

3.4. The reference period of an irregular wave field

Wave observations, necessary for the determination of the boundary conditions in a deltaic, i.e. shallow, area cannot always be obtained from the most favourable locations due to the high cost of the measuring stations; another detrimental factor is the usually limited observation time available for sampling. Consequently, boundary conditions must be derived from known wave conditions, measured on stations situated some distance away from the area concerned, by means of refraction computations. Theoretically, such refraction computations should be carried out separately for all components of a directional spectrum and for all circumstances of importance.

This approach is very laborious and often also inpractical, because of the limitations set by the accuracy of both the statistical evaluation of the available wave data and the final research results. Therefore, a representative wave celerity C_r or a representative period T_r must be defined in order to approximate the refraction computations for an irregular wave field, if a much more simplified computation method, viz. the refraction of the first order waves has to be utilized.

The mean period T_m as well as the statistical distribution of all "individual" periods of a wave field can be computed from wave records by the well known "zero crossing" method.

A comparison of schematized refraction patterns, obtained by radar with the computed refraction patterns in areas in which also simultaneous wave records are available shows the following results:

A. The mean period T_R determined from the mean wave lengths measured on the radar photographs and from the corresponding depths, appears to be a linear function of the mean period T_m computed from the corresponding time series: $T_R \approx 1,2\ T_m$ (fig.3.3). This result is of course only valid for that special type of radar and that particular position of the radar emmitor.

B. The best agreement between the observed and computed refraction patterns is obtained when this "radar period" T_r is used for the refraction computations.

The schematization of the wave pattern on a radar photograph is carried out manually and it is therefore not free of subjectivism. Results obtained from schematization of the same photograph by different persons, however, show only minor differences. The significant difference of T_r and T_m is caused by the geometry of the radar beam, because the smaller waves are invisible behind the larger ones.
The value of the coefficient, which is 1,2 in the above mentioned relation, depends on this geometry.
After discussing these results with the author, Ir. J.A. Battjes of the Delft Technological University derived a theoretical value of the coefficient from different types of spectra using the energy transport function of linear sinus waves as a weighting factor.

He found, that the values are in the range 1,2 ± 0,1 and explained the basis for the ressemblence between the observed and the computed refraction patterns using this coefficient.

The "radar period" correction must be determined by similar studies as mentioned above for each other types of radar or a different location of the emmitter.

Ir. L.A. Koelé of the Rijkswaterstaat studied the correlation between the wave heights and the periods of "individual waves" using a great number of time series from the North Sea (fig.3.4). His results which are not published, inspired the authors of the papers No. 3 and 10 of this Symposium to publish analogeous results of model experiments.

The correlation shows a statistically reliable correspondence between the significant wave height H_s and the period $T_r = (1,3 \pm 0,2)T_m$. However, the correlation is rather poor which fact is already stated by BRETSCHNEIDER (Ref.8).
This empirical result may be seen as an other argument for the application of the reference period $T_r = 1,2\ T_m$ in refraction computations.

4. DETERMINATION OF THE BOUNDARY CONDITIONS FROM A REFERENCE STATION.

The deeper insight in the problems concerning the irregular sea waves asked for better laboratory techniques. Also the demand for observations are changed qualitatively and quantitatively so as to provide the boundary conditions for the extensive engineering projects in coastal areas. However, the engineering decissions have to be based on long term observations and especially for this purpose the data from the wave gauges are usually insufficient.

Moreover, it is not possible to construct a great number of stations in the sea on account of both the cost involved and the time consuming elaboration of the data. For large projects such as the Delta project or the Europoort project a limited number of reference stations have been set up.

Long series of data may be obtained e.g. from the visual observations on light vessels and/or the visual observations performed on a board of merchant ships or by using the computed wave data from the long series of meteorological observations of wind velocity and direction.

Single visual observation of wave height should be treated with certain reservation because of the inherent subjective character. The comparison with instrumental wave gauge readings may give an insight in the reliability (fig.4.1). Analogeous arguments hold true for the value of the wave data computed from the observed wind conditions.

The statistical distributions of the parameters defining the irregular wave fields i.e. the significant wave heights H_s, the representative periods T_r and the mean directions of the wave propagation θ_m, may be of importance for the determination of the boundary conditions which are required for many engineering problems. They must be derived from a long period of observations. There are, however, not many stations in the world with series of wave recording longer than 5 years. Consequently, the data have to be extrapolated for the extreme conditions from rather few samples.

The distribution functions can almost never be described by formulae and the extrapolation becomes a very rough approximation especially because of the significancy lines of GUMBEL (Ref.9).

An example of the discrepancy between the statistical data on wave heights obtained from the Lightvessel "GOEREE" is given in fig.4.2 for two observation periods. It is obvious from this figure, that a mathematical solution of a decision problem concerning an engineering project usually does not show a very significant minimum of the sum of initial- and maintenance costs if it is based on such data.

** In the same figure the probability distributions of wave heights computed from visual observations and from readings obtained from the wave gauge "TRITON" (5 km offshore Scheveningen) are combined in order to determine the errors which might be obtained from the direct comparison of both distributions.

5. DETERMINATION OF THE CRITICAL DRAFT OF SHIPS IN A NAVIGATION CHANNEL

The depth below mean sea level of navigation channels in shallow waters determines the economy of many large harbours. The vastly increasing dimensions of the oil tankers and bulk-carriers call for still deeper channels. The costs of initial- and maintenance dredging increase more than proportionally to an increase in depth.

The physical factors which primarily influence the choice of the safe depth for ships of a certain mean draft are:

- tidal variation of the sea level
- wind-effect on the sea level
- roll, pitch and heave of the ship
- squat of the ship

A statistical method resulting in empirical probability function for the determination of the safe depth in the channel is described at the International Navigation Congress in 1964 (Ref.10). In that paper, the authors contribution deals with the empirical probability function for the combination of the first three factors using a mean response factor for determining the movement of the ship in a wave field (fig.5.1 and 5.2).

Economic decisions can be made by using these figures. The necessary depth of a channel can be determined provided the costs of initial dredging, the costs of maintenance dredging, the statistics on the density of navigation and their economical value for different types of ships are known.

The result of this three dimensional statistical analysis is the probability function which allows to determine the mean value of the risk of touching the bottom for a ship with a given mean response factor $\bar{\alpha}$ to the waves. The response factor α however is a function of three other main factors: the frequency of the wave-components of the energy density spectrum, the direction of the wave propagation in respect to the ship's course and speed of the ship.

The response factor could be applied to a Rayleigh distribution of wave heights in order to compute approximately the corresponding probability distribution of the deepest draft of the ship's.

The position of the deepest point changes continuously. It is determined as a sum of the three possible motions i.e. roll, pitch and heave.

The behaviour of large ships subjected to different types of wave fields are studied in a model basin in which it is possible to generate irregular waves.

Another study is undertaken in order to check the model results on a larger scale (approximately 1:5) in the North Sea using a self propelled barge with dimensions roughly corresponding to those of a tanker. It was impossible to use an actual tanker for experiments under the critical conditions because of the financial risks.

Some preliminary results of this research are given in fig. 5.3.

This research opens the possibility for refining of such conclusions which are already given in the paper mentioned above. The risks of touching bottom for the largest ships, which want to approach the harbour under critical conditions, can be eliminated by the decision that the ship have to remain in deep water.

The economical loss caused by a delay of a tanker and the probability of the delay of a certain duration is a determining factor when loading the ships. The danger of an accident like the well known grounding of Torry Canion may very well determine the level of risks to be permitted by the harbour authorities. For both reasons, operational guidance is necessary. This guidance should be based on an on-line analysis of wave fields and water

levels and on a short time prediction of both.
The following method is in development for this purpose:

A. On line analysis of the record of a permanent wave station near the navigation channel will give an energy density spectrum of waves which is representative for the data sampling period depending on the importance of the lowest frequency components. This sampling period is about 30 min for the North Sea, so the mean value of the spectrum is obtained with a delay of 15 min. The mean direction of propagation of the wave fields will be derived from radar-observations.

B. Continuous registration of filtered variations of the sea-level will give the corresponding value of the reduction of the recente soundings in order to get the critical depth in the channel.

C. The energy density spectrum of the ships movement will be computed for the types of ships approaching the channel. The following data are necessary: the speed, the course and the type of the ship concerned and the response functions of the ship's movements corresponding with these data. It is to be expected, that there will be only a limited number of types of the very large ships. The companies concerned will provide these necessary data when detrimental effect and economic loss of eventual delay's become apparent.

D. The probability distribution of the maximum draft of the ship will be determined from the spectrum of the ship's movement, corrected by squat. The squat may be approximated by using the data mentioned above.

E. The check on the danger-probability level by comparing the results of the computations with the accepted risks-criteria determine the time necessary for warning the ships commander. The expected change in weather conditions and the state of sea must also be considered by cooperation with a meteorological service, and the expected delay can be passed to the commander.

6. BOUNDARY CONDITIONS ON WAVES IN DESIGN OF A DIKE

The sea dikes of the modern type in the Delta are designed so as to allow a limited overtopping by waves under conditions which never were observed before. These conditions are determined by the critical storm defined by the critical storm surge level. This level is the result of an economical decision study. The mean probability of occurrence of this storm or of a higher one within a century is fixed to the value of 1%.

Only 2% of the wave tongues are accepted to overtop the dike during the above critical stormconditions.

Delft Hydraulic Laboratory carried out experiments in the wind flume in which certain boundary conditions, heights and profiles of dikes were used.

Hence the choice of the design profile is strictly related to certain critical boundary conditions which, as we hope, will never be experienced in the nature. Moreover, the boundary conditions will change in the course of time by the changing morphology of the area to the seaward of the dike. The design of the dike and especially the determination of the height is almost impossible without further schematization of the problem.

The following assumptions were made when determining the profile of the dike in one of the delta estuaries i.e. "Brouwershavense Gat".

A. The mean direction of propagation of the waves on the North Sea is determined by the expected direction of the critical storm wind, which is North-West. The corresponding water level with the above mentioned exceedance probability is in this area about 5 m above mean sea lev l.

B. The waves in the estuary are smaller than the sea waves because of breaking on the outlaying shallow banks. The significant wave height is limited to: $H_s \approx 0{,}6$ d which is about 30% too high as compared to recent observation data which were not available at the time when the decision on the design of the dike had to be made (1964).

C. The wave height is further determined by the refraction coefficient computed from refraction computations using the reference period $T_r = 12$ sec. This period is probably too long according to our present knowledge.

D. The critical significant wave height can be approximated by $H_s = \sqrt{\Sigma H^2_k}$ in regions in which two or more significantly different directions of wave propagation Θ_k occur. H_k is the significant wave height corresponding with the direction Θ_k, k = 1,2,.....

E. The boundary condition is determined at the distance of about 200 m of the dike. This is the distance, which can be reproduced on scale in most of the laboratory flumes. Moreover, at this distance the reflection of irregular waves does not significantly affect the wave pattern induced by the wave generator.

The results of the construction of wave rays is shown in fig. 6.1. The significant wave heights between neighbouring rays in deep water is computed along their tracks using the theory of constant energy transport. Corrections of this significant wave height H_s are computed for each point where the depth is less then $0{,}6\ H_s$ (Assumption B). For the determination of the boundary condition i.e. the significant wave height 200 m out of the dike, the principle of superposition is applied (Assumption D).

In fig.6.2, the significant wave height along the dike is shown (graph A). In order to check the breaking condition for the total result after superposition, the maximal significant wave height $H_s = 1{,}3$ d which is possible for depths along the line 200 m out of the dike is determined (graph B). For most of the points of the dike two values can be obtained from these graphs; the lowest value found is used for the critical significant wave height at that particular spot.

** The value $H_s = 5$ m is chosen to be the representative boundary condition for whole the dike.

A second group of computations were carried out based on the possible changed bottom conditions. This change is expected after the estuary has been closed. A comparison of the results showed that the critical significant wave heights in future cannot be higher than the chosen boundary conditions.

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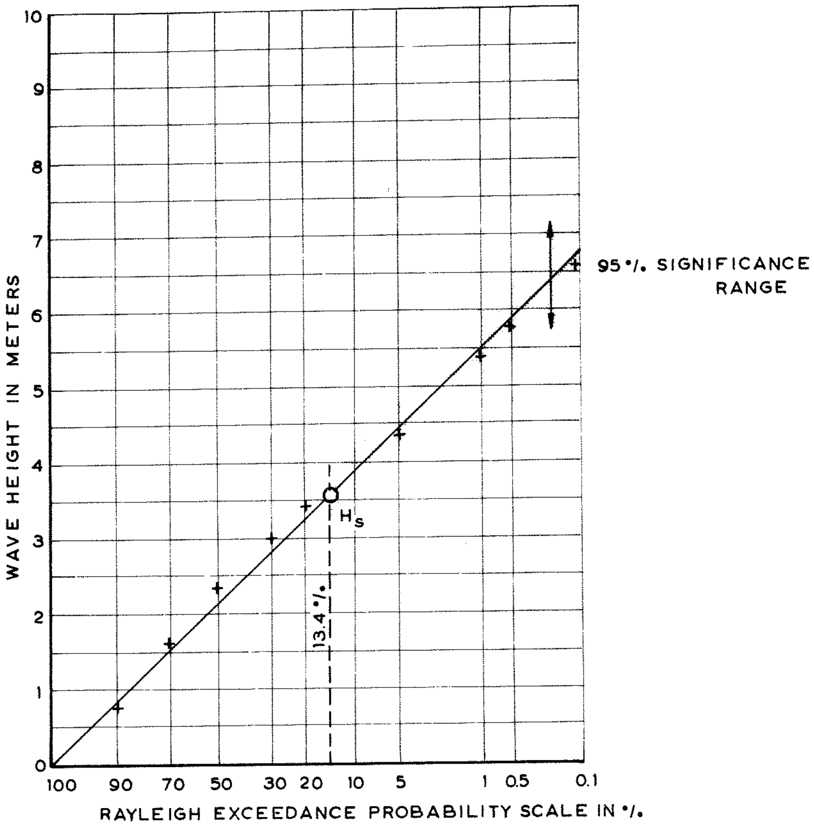


Fig. 3.1 EXAMPLE OF MEAN EXCEEDANCE PROBABILITY OF „INDIVIDUAL WAVES" OF A RECORD IN 16 m DEPTH

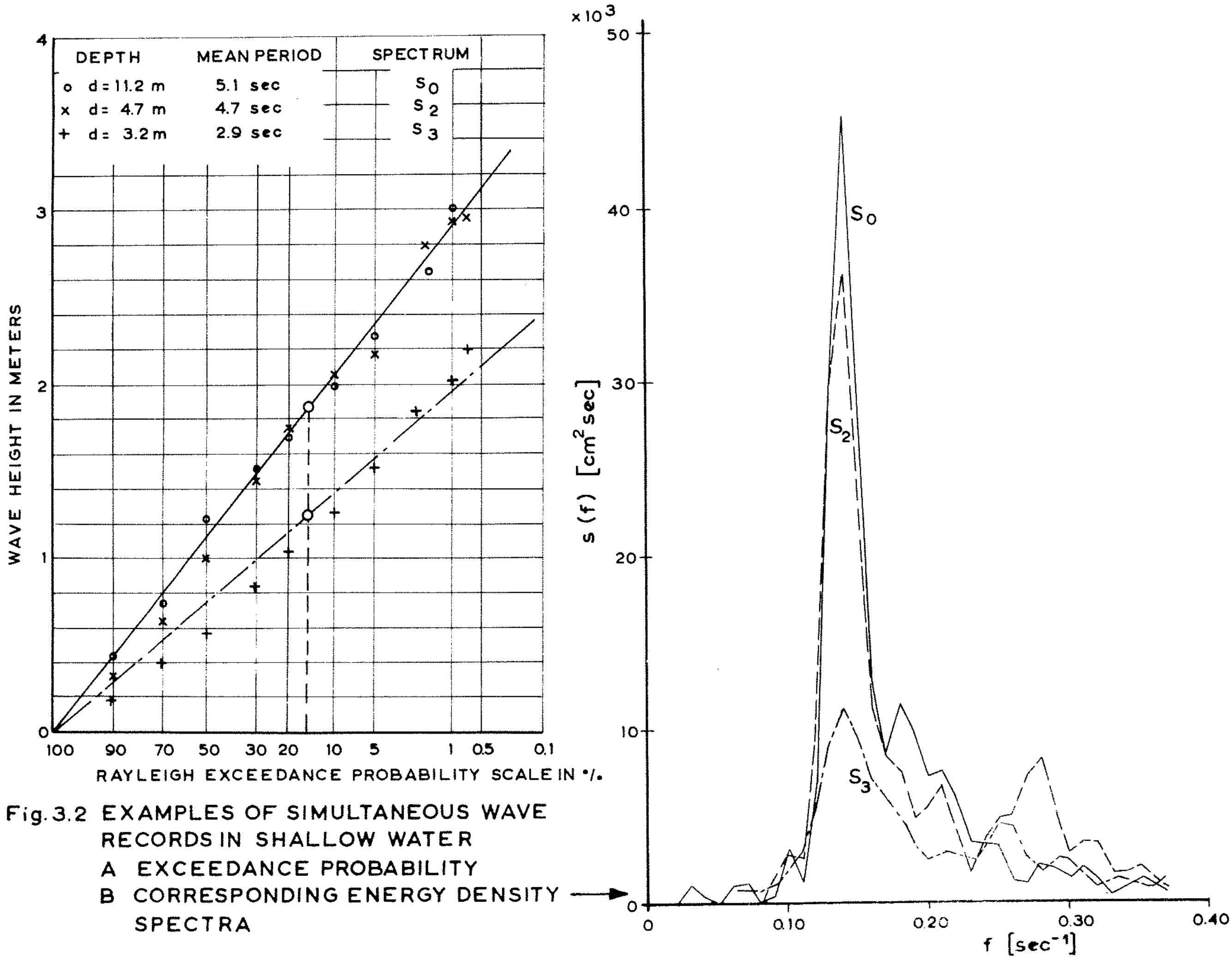


Fig. 3.2 EXAMPLES OF SIMULTANEOUS WAVE RECORDS IN SHALLOW WATER
A EXCEEDANCE PROBABILITY
B CORRESPONDING ENERGY DENSITY SPECTRA

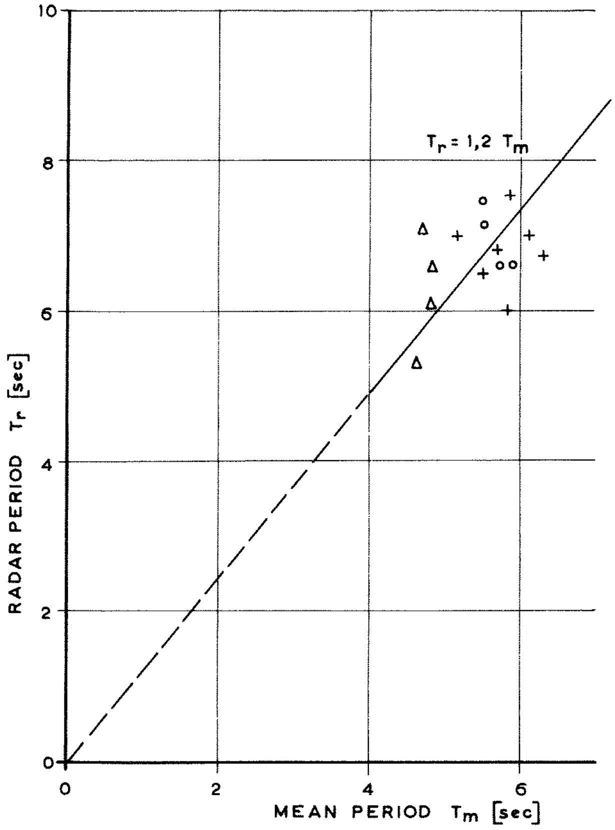


Fig. 3.3 COMPARISON BETWEEN „ RADAR PERIOD" T_r USED FOR REFRACTION COMPUTATIONS AND MEAN PERIOD T_m FROM TIME SERIES

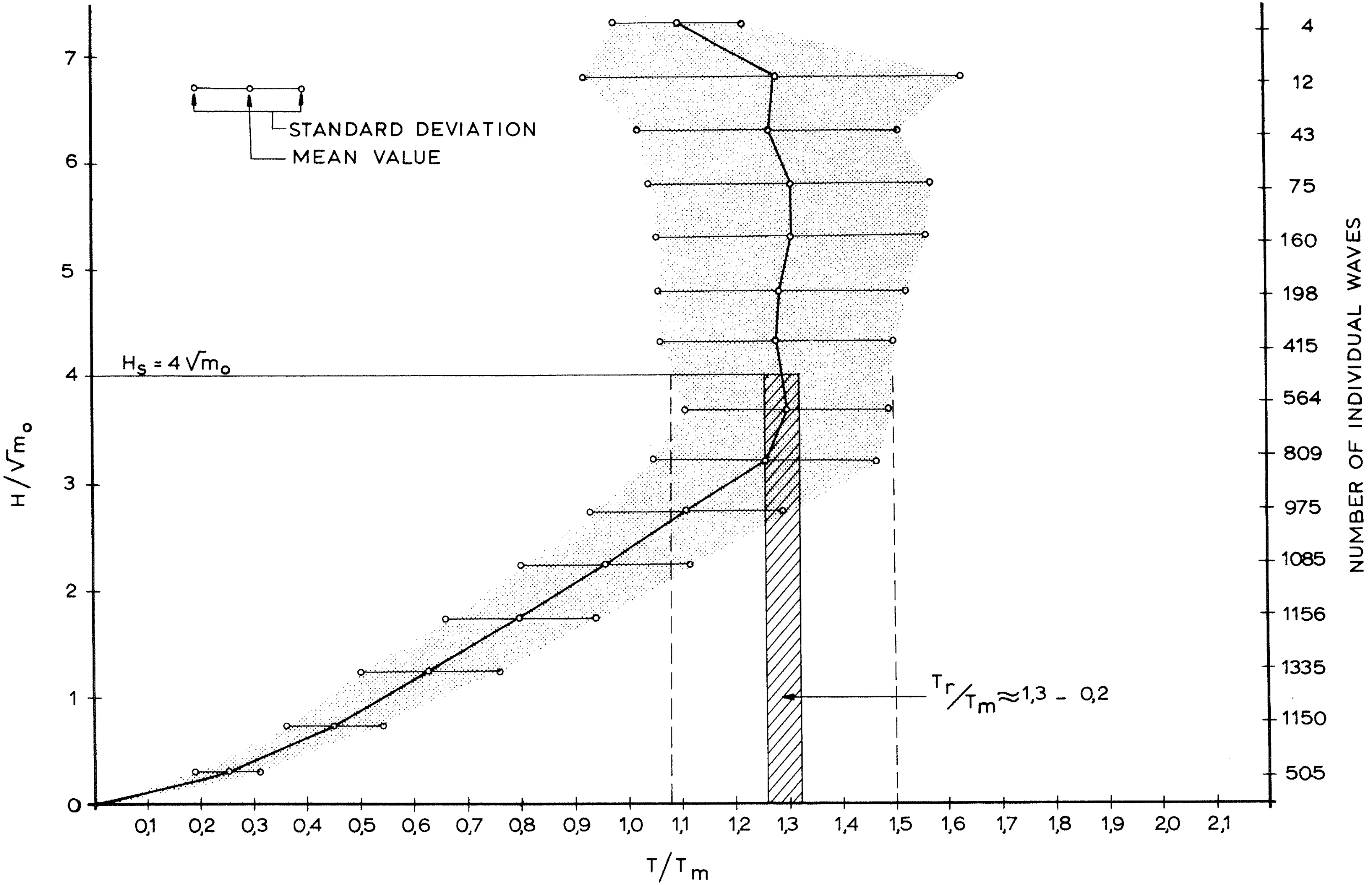
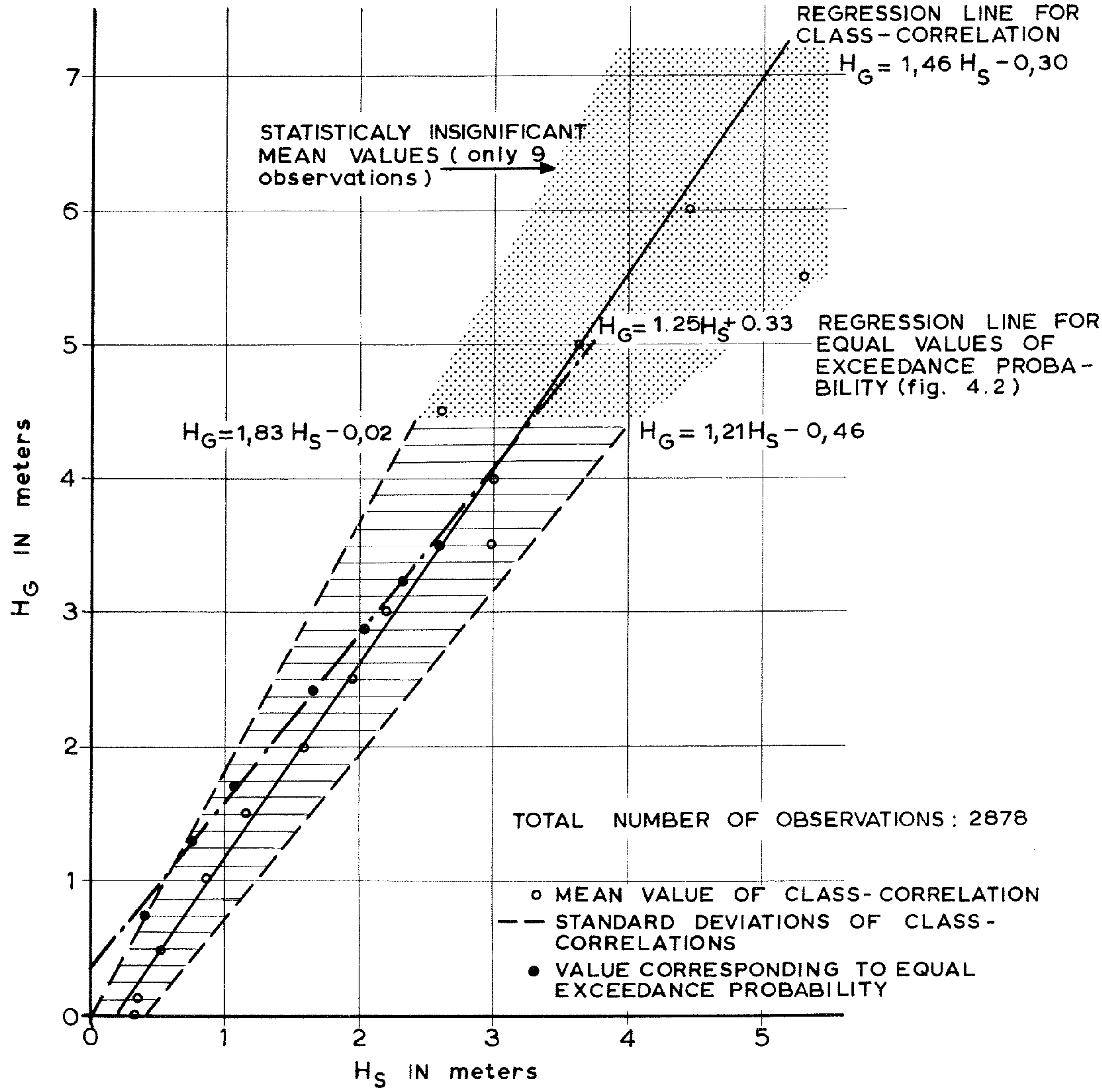


Fig. 3.4 CORRELATION BETWEEN THE HEIGHT H AND PERIOD T OF "INDIVIDUAL WAVES" ON THE NORTH SEA



* Fig. 4.1 CORRELATION BETWEEN VISUAL OBSERVATIONS OF WAVE HEIGHTS H_G (LIGHT VESSEL „GOEREE") GROUPED IN CLASSES OF 0,5 m AND SIGNIFICANT WAVE HEIGHTS H_S OBTAINED FROM RECORDS AT „TRITON" STATION

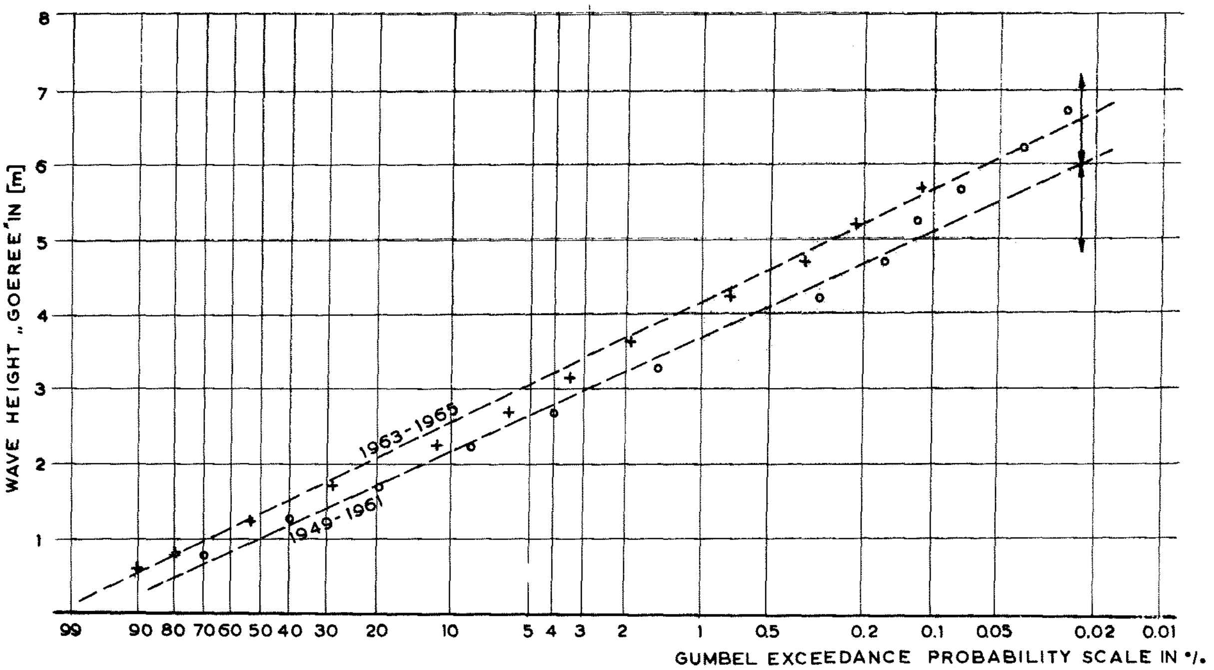
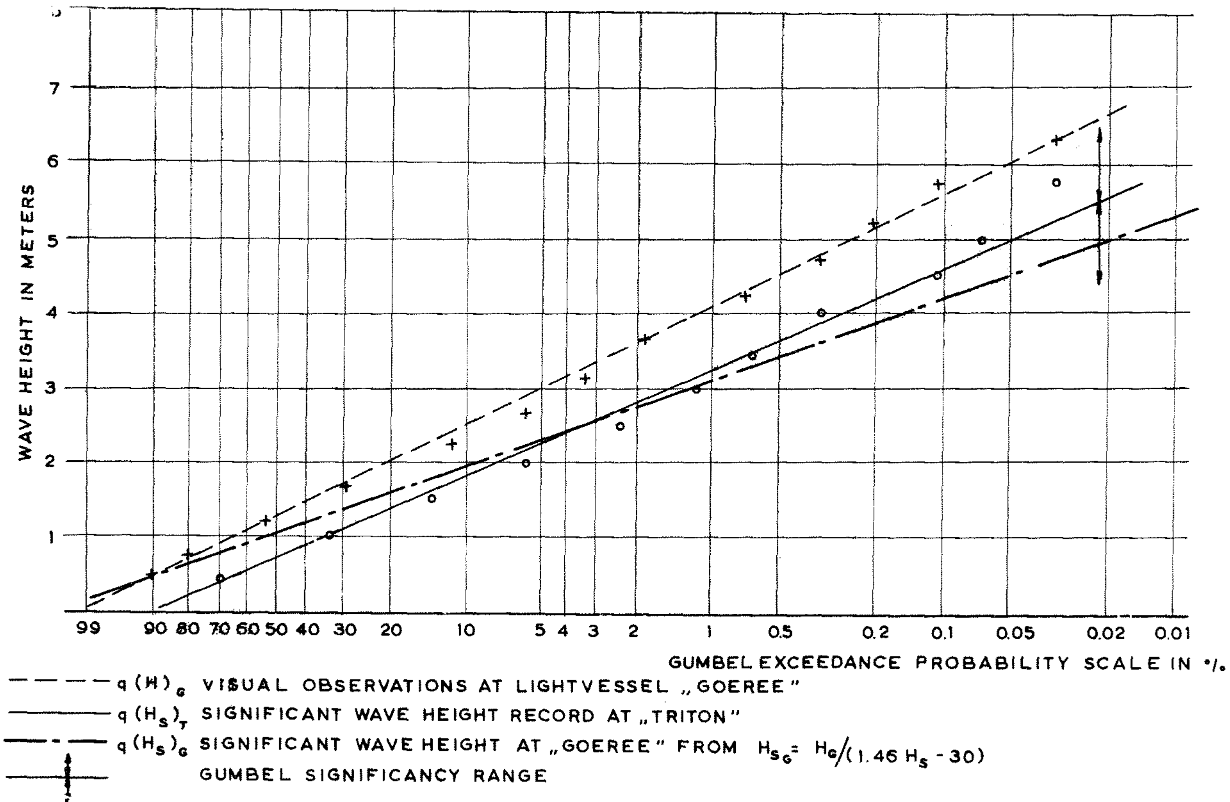
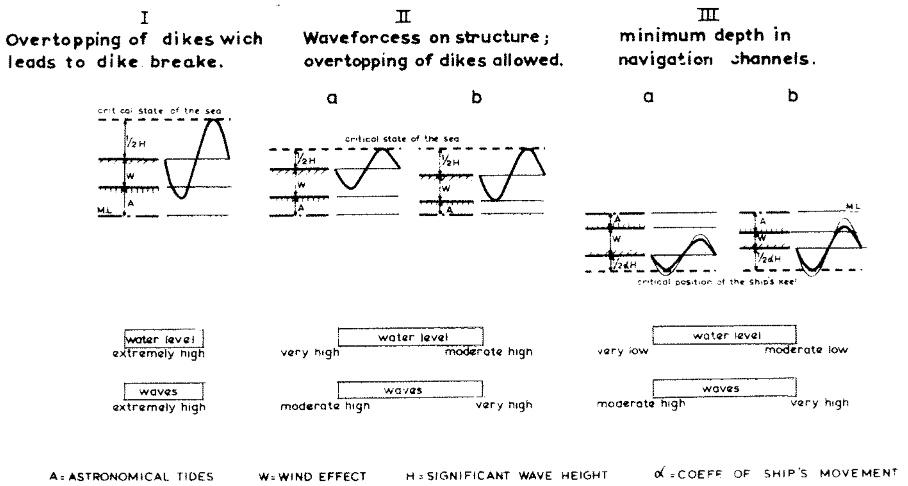


Fig. 4.2a COMPARISON BETWEEN TWO PERIODS OF VISUAL OBSERVATIONS AT LIGHTVESSEL GOEREE



* Fig. 4.2 b REDUCTION OF EXCEEDANCE PROBABILITY FROM CORRELATION SHOWN IN Fig 4.1.



* Fig 5.1 EXAMPLES OF CRITICAL CONDITIONS OF THE SEA – SURFACE

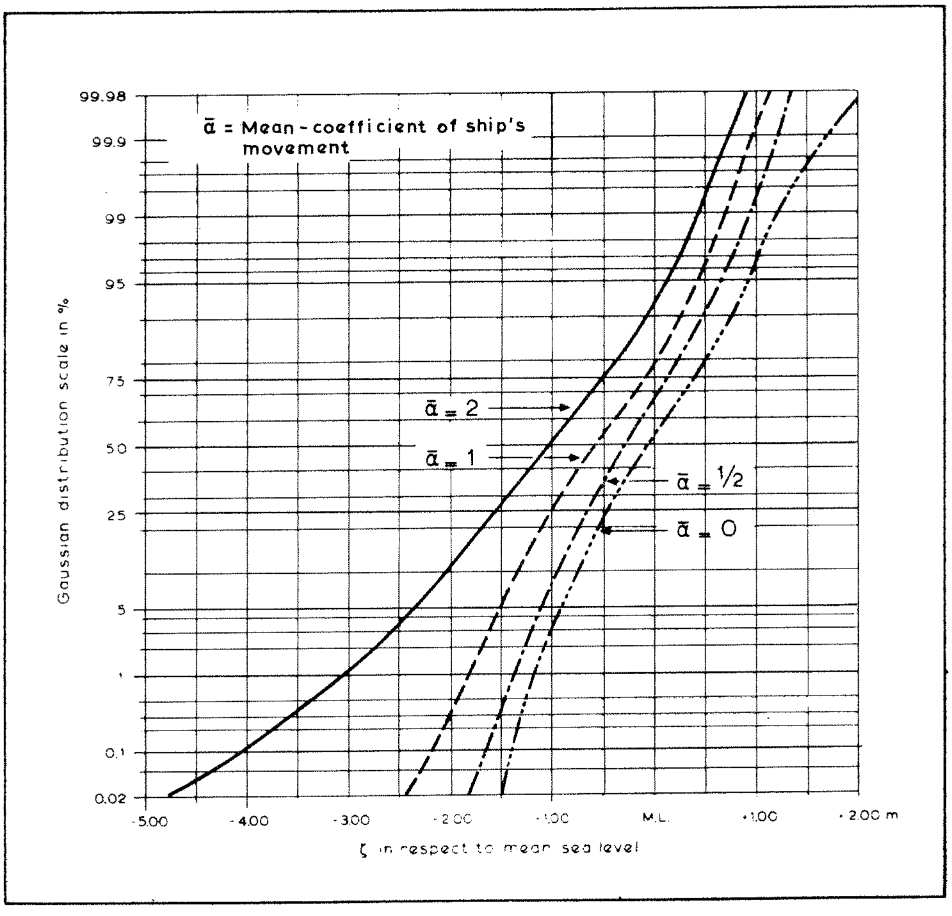


Fig. 5.2 RESULTING CUMULATIVE PROBABILITY DISTRIBUTIONS OF THE LOWEST POSITION ζ OF SHIP'S KEEL IN RESPECT TO MEAN SEA LEVEL M.L.

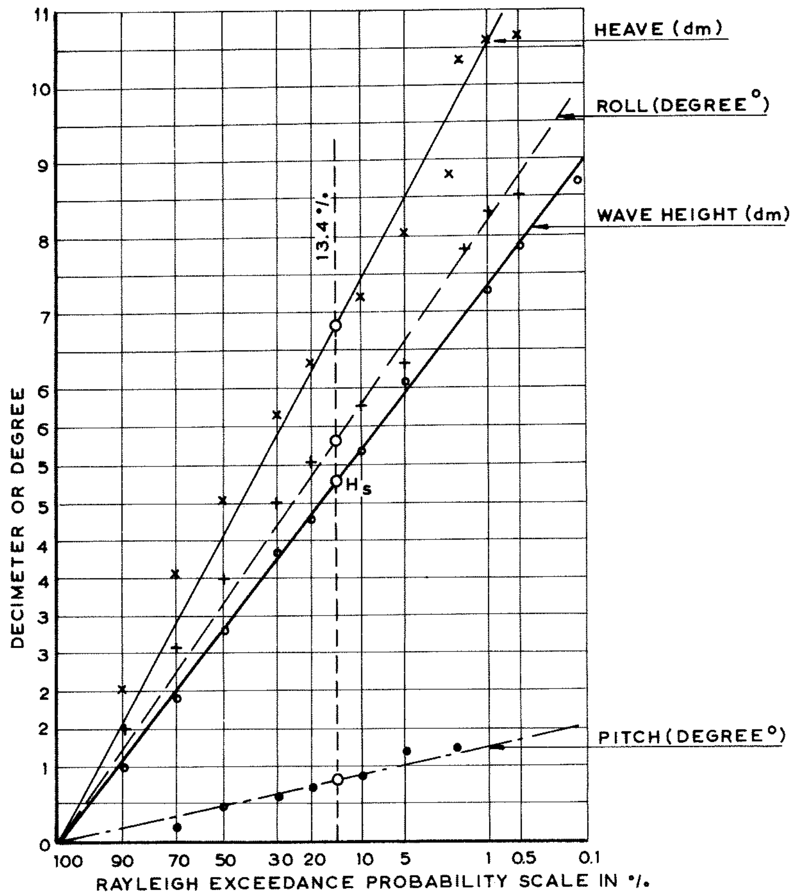


Fig. 5.3 COMPARISON BETWEEN SHIPS MOVEMENT AND RECORDED WAVES

REFRACTIEPATTERN FOR BROUWERSHAVENSE GAT

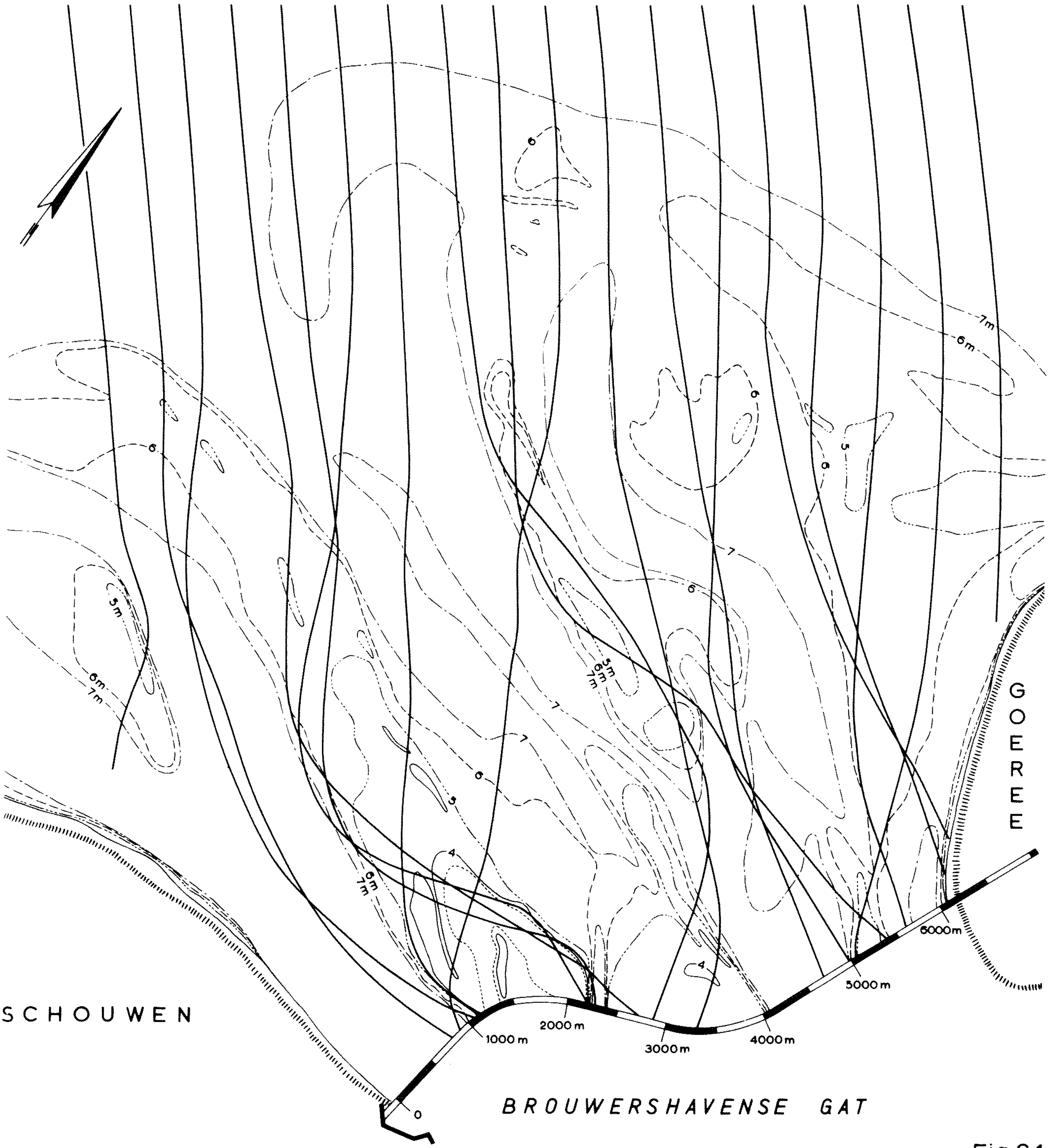


Fig. 6.1

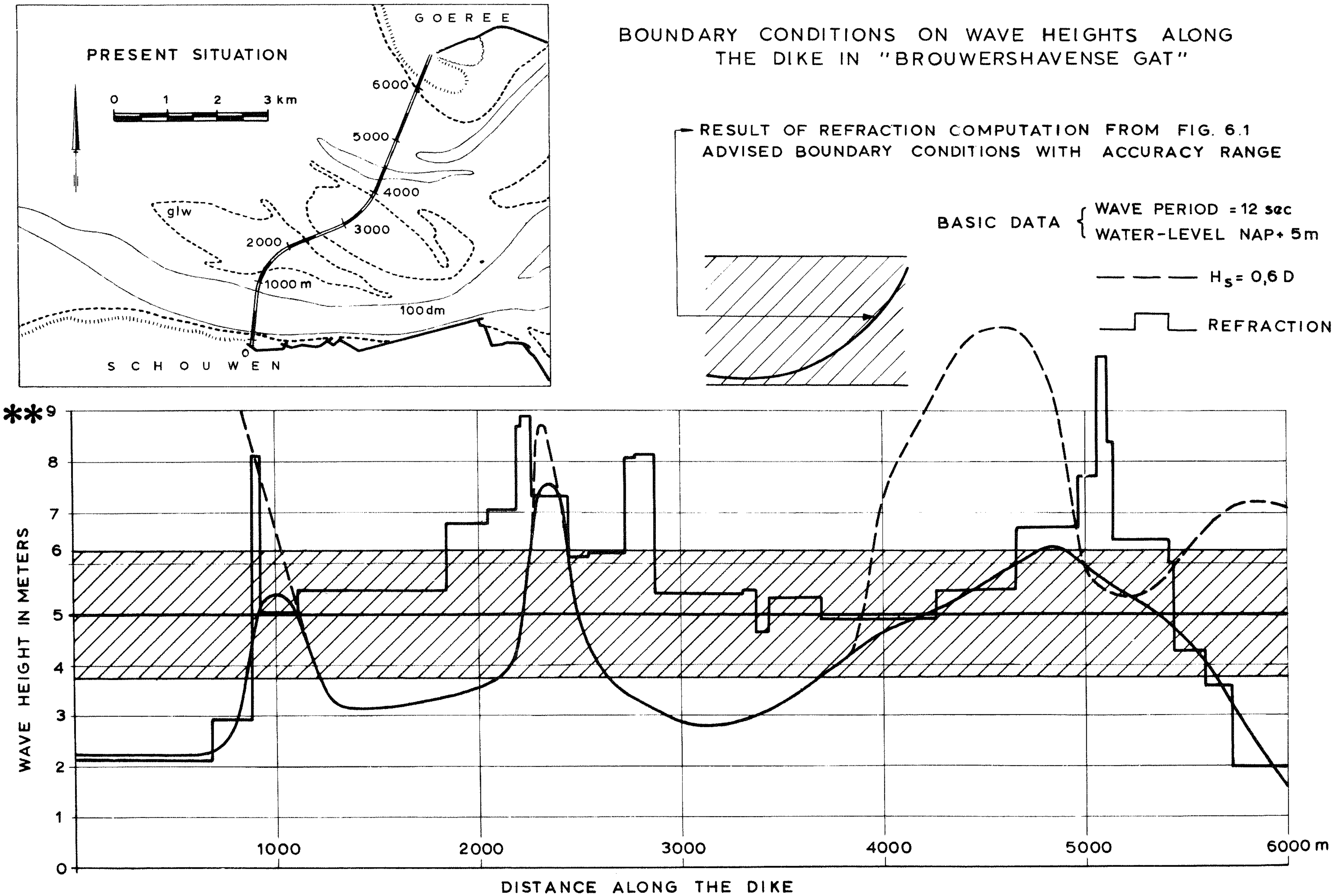

BOUNDARY CONDITIONS ON WAVE HEIGHTS ALONG THE DIKE IN "BROUWERSHAVENSE GAT"
PRESENT SITUATION
0 1 2 3 km
GOEREE
SCHOUWEN
glw
100dm
0
1000 m
2000
3000
4000
5000
6000
RESULT OF REFRACTION COMPUTATION FROM FIG. 6.1
ADVISED BOUNDARY CONDITIONS WITH ACCURACY RANGE
BASIC DATA
WAVE PERIOD = 12 sec
WATER-LEVEL NAP+ 5m
H s = 0,6 D
REFRACTION
**
WAVE HEIGHT IN METERS
0
1
2
3
4
5
6
7
8
9
1000
2000
3000
4000
5000
6000 m
DISTANCE ALONG THE DIKE

Fig. 6.2

DISCUSSION ON PAPER 1

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It is pointed out by Svašek that in certain computations the energy of a random wave field, with a continuous spectrum, is lumped at a single frequency. This procedure is sometimes applied in problems of refraction, even though this is a frequency-dependent phenomenon for the waves being considered. However, neither the shortcomings nor the merits of the procedure will be discussed here. The purpose of this discussion is merely to provide a criterion which may be used in the determination of an "equivalent" monochromatic wave train, and to give expressions for the "equivalent" height and period resulting therefrom. The criterion which has been adopted here states that in deep water the monochromatic wave shall have the same average energy and energytransport as the random wave field. The term "equivalent" refers to this condition only and in no way implies the correctness of the simplifying procedure.

At first, waves which are long-crested in deep water will be considered. Let $S(\omega)$ be the energy spectrum, with moments

**

$$m_j = \int_0^\infty \omega^j S(\omega)\, d\omega \qquad (1)$$

The energy content per unit area is $\rho g m_0$, and the equivalent height of the monochromatic wave is given by

$$\frac{1}{8} H_{eq}^2 = m_0 \qquad (2)$$

The condition of equal energy transport may be written as

$$\frac{1}{8} H_{eq}^2\, c_{g_{eq}} = \int_0^\infty c_g(\omega)\, S(\omega)\, d\omega \qquad (3)$$

or

$$c_{g_{eq}} = \frac{\int_0^\infty c_g(\omega)\, S(\omega)\, d\omega}{\int_0^\infty S(\omega)\, d\omega} \qquad (4)$$

The equivalent group velocity appears as the average group velocity of the spectral components, weighted with the spectral density $S(\omega)$.

In deep water, $c_g = \frac{g}{2\omega} = gT/4\pi$. It then follows from (4) that the equivalent period is given by

$$T_{eq} = 2\pi \frac{m_{-1}}{m_0} \qquad (5)$$

It is convenient to compare this value with T_m, the mean zero up- or down-crossing period:

$$T_m = 2\pi \sqrt{\frac{m_0}{m_2}} \qquad (6)$$

Thus

$$\frac{T_{eq}}{T_m} = \frac{m_{-1}\, m_2^{1/2}}{m_0^{3/2}} \qquad (7)$$

If

$$S(\omega) = a\,\omega^{-b}\, e^{-c\omega^{-d}} \qquad (8)$$

then

$$\frac{T_{eq}}{T_m} = \frac{\Gamma\left(\frac{b}{d}\right)\Gamma^{1/2}\left(\frac{b-3}{d}\right)}{\Gamma^{3/2}\left(\frac{b-1}{d}\right)} \qquad (9)$$

Numerical values are given in Table 1 for a spectrum which is similar to a Neumann spectrum, a Bretschneider spectrum or a Pierson-Moskowitz spectrum.

	b	d	T_{eq}/T_m
Neumann	6	2	1.23
Bretschneider Pierson-Moskowitz	5	4	1.20

Table 1

It is possible to carry out an analogous averaging procedure for waves which are short-crested in deep water. The group velocities in Eqs. (3) and (4) should then be treated as vectors. For a numerical estimate the energy is assumed to be confined to components travelling within ± 90° from the resultant direction $\alpha = 0°$, and to be distributed according to $\cos^n \alpha$. Eq. (7) is then replaced by

**

$$\frac{T_{eq}}{T_m} = f(n)\, \frac{m_{-1}\, m_2^{1/2}}{m_0^{3/2}} \qquad (10)$$

in which

$$f(n) = \frac{\int_0^{\pi/2} \cos^{n+1}\alpha \, d\alpha}{\int_0^{\pi/2} \cos^{n}\alpha \, d\alpha} \qquad (11)$$

Common values of n range from 4 to 10, with a corresponding range of $f(n)$ from 0.91 to 0.96. Thus, if the averaging procedure is applied not only to the frequencies but also to the directions, the values of T_{eq}/T_m listed in Table 1 should typically be reduced by 5% to 10% approximately.